

Yo-Yo PHYSICS:

AN ENGINEER'S NOTEBOOK

Yo-Yo RPM

MONOGRAPH V

IN A SERIES IV

IN A SERIES

Don Watson

10/2003

Captain Yo

AUTHOR'S NOTE:

POPULAR HIGH SCHOOL AND COLLEGE LEVEL TEXT BOOKS ON PHYSICS AND MATHEMATICS ARE THE MAJOR SOURCES FOR DEVELOPMENT OF THIS MONOGRAPH SERIES. OTHER SOURCES INCLUDE MANY PUBLICATIONS IN MY PERSONAL LIBRARY; SEE "REFERENCES" IN EACH MONOGRAPH.

1630 - THE NUMERIC YO-YO CONSTANT, AND THE "SLEEPER" EQUATIONS ARE MY OWN DERIVATIONS, FULLY DETAILED HERE USING THE CLASSIC RELATIONSHIPS GIVEN IN MANY STANDARD REFERENCES FOR ENERGY OF TRANSLATION AND ENERGY OF ROTATION FOR MASSES IN MOTION.

MUCH EFFORT HAS BEEN EXPENDED IN CHECKING AND RECHECKING THE WORK. IF ERRORS OF ANY KIND REMAIN, THEY ARE SOLELY MINE.

YOURS FOR YO-YOS OF THE FUTURE -

DW
9/03

YO-YO PHYSICS:
AN ENGINEER'S NOTEBOOK

YO-YO RPM
MONOGRAPH II
IN A SERIES

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REFERENCES - INSIDE BACK COVER

INTRODUCTION

MANY CHANGES HAVE OCCURRED IN THE YO-YO TRADE AND HOBBY DURING THE PAST TWENTY YEARS. YO-YOS SPIN LONGER AND ARE MORE STABLE; MANY MORE MANUFACTURERS SUPPLY THEM; THE INTERNET SUPPORTS NEW MARKETING OPTIONS, CLUBS, ORGANIZATIONS, AND PLAYER COMMUNICATION; ADVANCES IN SKILL LEVELS ARE ASTONISHING, ESPECIALLY IN THE PAST DECADE; POPULAR STYLES OF PLAY NOW INCLUDE "FREE HAND" AND "OFF STRING".

IN THE OLD DAYS OF FIXED WOOD AXLES, WITH YO-YOS SHAPED TO FIT THE HAND, SPIN DURATIONS WERE LIMITED; 10 OR 15 SECONDS BEING A GOOD ACCOMPLISHMENT FOR MOST PLAYERS. TODAY, WITH BALL BEARING AXLES, SPIN DURATIONS OF A FEW MINUTES ARE COMMON FOR MANY PLAYERS. ROLLING FRICTION IN THE BEARING, REPLACING STRING-TO-WOOD CAPSTAN FRICTION, MAKES THE DIFFERENCE. GREATER WEIGHT WITH PROMINENT RIM WEIGHTING IN MANY NEW DESIGNS PROVIDES GREATER ANGULAR (SPINNING) MOMENTUM TO TAKE ADVANTAGE OF THE LOWER AXLE FRICTION - ADDING SIGNIFICANTLY TO STABILITY AND SPIN DURATION.

DESPITE COINCIDENT LOSS IN INITIAL "SLEEPER" RPM,

"SLEEPER" RPM STUDIES HERE BEGIN WITH ENERGY ANALYSIS FOR A TETHERED YO-YO IN FREE FALL FROM THE HAND TO DETERMINE INITIAL "SLEEPER" ANGULAR VELOCITY REACHED AT THE END OF THE STRING.

A NUMERIC YO-YO CONSTANT RELATING INITIAL "SLEEPER" RPM DIRECTLY TO THE YO-YO RADIUS OF

GYRATION IS DEFINED FOR THE TETHERED FREE FALL CASE, WITH MEASURED AND CALCULATED DATA PROVIDED IN CHART AND GRAPH FORM FOR FIFTEEN YO-YO VARIATIONS INCLUDING SEVERAL POPULAR COMMERCIAL MODELS. EACH COMMERCIAL MODEL LISTS THE MANUFACTURER AND MODEL NAME (THE LATTER IN "QUOTES"); EACH MAY HAVE DIFFERENT PLAYING CHARACTERISTICS, BUT ALL ARE EXCELLENT YO-YOS FOR ANY SERIOUS PLAYER TO OWN.

AN ENERGY ANALYSIS IS DEVELOPED FOR A "SLEEPER" THROW WITH THE YO-YO ASSUMED TO LEAVE THE HAND AT AN ASSUMED VERTICAL VELOCITY; HERE "SLEEPER" EQUATIONS ARE DEVELOPED AND SPIN DURATION IS EVALUATED. YO-YO RPM MEASUREMENT WITH TACHOMETERS AND STRING COUPLING (OR LACK OF IT) ARE DISCUSSED. FINALLY, TURNING PRECESSION NOW STRONGLY EVIDENT IN MOST WIDE STRING GAP YO-YOS IS ANALYZED.

I OWE REAL APPRECIATION TO YO-YO FRIENDS JOHN STANGLE, BILL DEBOISBLANC, DAVID CAPURRO, AND HAL GRIFFIN FOR THEIR HELP IN TACHOMETER TRIALS WITH SEVERAL YO-YOS; SPECIAL THANKS TO JOHN AND TO LAURI EDGAR FOR HOSPITALITY AT THEIR HOME. THANKS ALSO TO NATIONAL YO-YO GRAND MASTER "BDEB" FOR MANY HELPFUL DISCUSSIONS AND FOR HIS VALUABLE SUGGESTION THAT A YO-YO "SLEEPER" THROW IS ANALOGOUS TO A BASEBALL THROW WHERE PROFESSIONAL BALL PLAYERS ARE KNOWN TO PITCH "FASTBALLS" REGULARLY AT AND AROUND 100 MPH.

YO-YO RPM - AN ENERGY APPROACH

AN UNTETHERED YO-YO OF WEIGHT M , STARTING AT REST AND UNDER THE INFLUENCE OF GRAVITY ACCELERATION g , ENDS A FALL OF DISTANCE S WITH KINETIC ENERGY $KE_s = Mgs$. IN THE FREE FALL, KE_s APPEARS AS ENERGY OF TRANSLATION TE_s (LINEAR MOTION). THE YO-YO, LACKING AN UNWINDING TETHER, DEVELOPS NO ENERGY OF ROTATION RE_s . A YO-YO WITH $M = 45 \times 10^{-3} \text{ kg}$ FALLING FREELY UNDER GRAVITY $g = 9.81 \text{ m/sec}^2$ A DISTANCE $S = 1.0 \text{ m}$, DEVELOPS KE_s AT END OF FALL AS:

$$KE_s = TE_s = Mgs \text{ kg} \cdot \text{m}^2/\text{sec}^2 \text{ OR } * \text{N} \cdot \text{m}$$

$$KE_s = TE_s = 45.0 \times 10^{-3} \cdot 9.81 \cdot 1.0 = 0.441 \text{ N} \cdot \text{m}$$

THE SAME YO-YO TETHERED, RELEASED FROM THE HAND AT (AND REMAINING AT) REST, AND UNWINDING ITS STRING WHILE FALLING UNDER THE INFLUENCE OF GRAVITY ALONE, TAKES LONGER TO FALL $S = 1.0 \text{ m}$ BUT DEVELOPS THE SAME $KE_s = 0.441 \text{ N} \cdot \text{m}$. HERE, KE_s HAS TWO COMPONENTS: ENERGY OF TRANSLATION TE_s AND ENERGY OF ROTATION RE_s . AT $S = 1.0 \text{ m}$, WE CAN WRITE:

$$KE_s = TE_s + RE_s = 0.441 \text{ N} \cdot \text{m}$$

* 1 Newton IS THAT FORCE ACCELERATING A MASS OF 1 kg AT 1 m/sec^2 ; $1 \text{ N} = 1 \text{ kg} \cdot \text{m/sec}^2$

AT END-OF-FALL (OR A SLIGHT INSTANT EARLIER) THE VERTICAL (LINEAR) VELOCITY V_s m/sec AND AN-
GULAR (ROTATING) VELOCITY ω_s rad/sec ARE
DIRECTLY RELATED AS $\omega_s = V_s/r$ WHERE r IS THE
AXLE RADIUS IN METERS. GIVEN THE Y_0-Y_0 ROTAT-
ING MOMENT OF INERTIA J kg-m²:

$$TE_s = \frac{1}{2} M V_s^2; RE_s = \frac{1}{2} J \omega_s^2 = \frac{1}{2} J \left(\frac{V_s^2}{r^2} \right)$$

ELIMINATING V_s^2 :

$$V_s^2 = \frac{2TE_s}{M} = \frac{2RE_s r^2}{J}; RE_s = TE_s \left(\frac{J}{M r^2} \right)$$

USING $J = M k_o^2$, OR $k_o^2 = \left(\frac{J}{M} \right)$ WHERE k_o IS THE
 Y_0-Y_0 RADIUS OF GYRATION:

$$RE_s = TE_s \left(\frac{k_o^2}{r^2} \right); TE_s = RE_s \left(\frac{r^2}{k_o^2} \right)$$

AND FROM THE PRECEDING PAGE:

$$KE_s = TE_s + RE_s = RE_s \left(\frac{r^2}{k_o^2} + 1 \right) = 0.441 \text{ N-m}$$

WOLFGANG BÜRGER (1984) DEFINED A SAMPLE
 Y_0-Y_0 WITH AN AXLE RADIUS OF 0.3 CM OR $r = 3 \times 10^{-3}$ m
AND A RADIUS OF GYRATION "OF ABOUT 2.5 CM" OR
 $k_o = 2.5 \times 10^{-3}$ m. MONOGRAPH IV ASSIGNED THAT
"PLAYABLE Y_0-Y_0 " THE WEIGHT $M = 45 \times 10^{-3}$ kg (WITH
THE RESULTING MOMENT OF INERTIA $J = 28125 \times 10^{-9}$
kg-m²). USING THESE KNOWN VALUES WITH $KE_s =$
0.441 N-m, CALCULATE RE_s :

$$RE_s = KE_s / \left(\frac{r^2}{k_o^2} + 1 \right) = 0.441 / \left[\left(\frac{3 \times 10^{-8}}{25 \times 10^{-2}} \right)^2 + 1 \right]$$

$$RE_s = 0.441 / \left(\frac{9}{625} + 1 \right) = 0.441 / 1.014$$

$$RE_s = 0.435 \text{ kg-m}^2/\text{sec}^2 \text{ OR N-m}$$

END-OF-FALL ANGULAR VELOCITY RPM_{EOF} IS NOW EASILY DETERMINED FROM $RE_s = \frac{1}{2} I \omega_s^2$ WHERE:

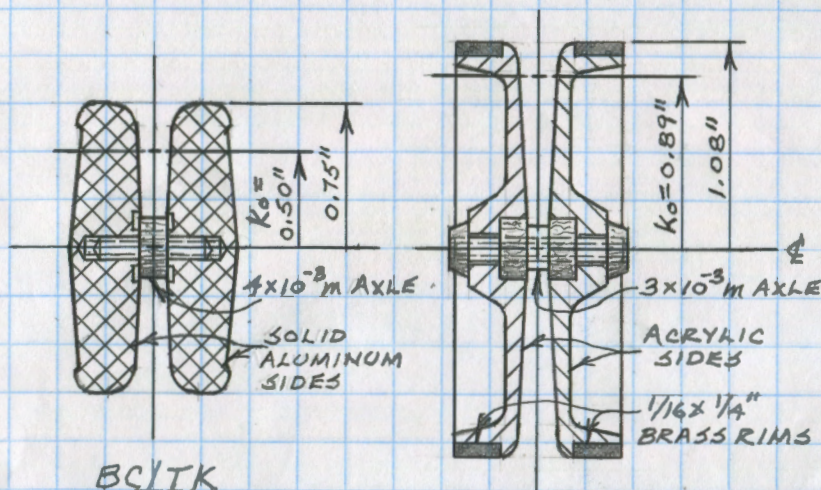
$$\omega_s = \left(\frac{2 RE_s}{I} \right)^{\frac{1}{2}} = \left(\frac{2 \cdot 0.435}{28125 \times 10^{-9}} \right)^{\frac{1}{2}} = \left(\frac{2 \cdot 4.35 \times 10^8}{28125} \right)^{\frac{1}{2}}$$

$$\omega_s = 176 \frac{\text{rad}}{\text{sec}} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{60 \text{ sec}}{\text{min}}$$

$$RPM_{EOF} = \omega_s = 1681 \text{ rev/min}$$

THIS YO-YO, AND YO-YOS GENERALLY, DEVELOP ENERGY OF ROTATION VERY EFFICIENTLY. IN THIS CASE, RE_s IS 98.6% OF END-OF-FALL KE_s .

YO-YOS ON THE NEXT PAGE MAKE THE POINT THAT RPM_{EOF} VARIES DIRECTLY WITH ENERGY OF ROTATION RE_s AND INVERSELY WITH MOMENT OF INERTIA I . THE "POCKET ROCKET", SMALL IN SIZE AND WITH NO APPARENT RIM-WEIGHTING ACHIEVES A SURPRISINGLY HIGH RPM_{EOF} . THE CAPTAIN YO MODEL, BUILT IN 1987 (NON-COMMERCIAL) TO EVALUATE EXTREME RIM-WEIGHTING WITH LIGHT PLASTIC HALVES AND BRASS RIMS, ACHIEVES A MUCH LOWER RPM_{EOF} . NOTE THAT THESE TWO YO-YOS ARE OF VERY NEARLY EQUAL WEIGHT $M \approx 48 \times 10^{-3} \text{ kg}$.



POCKET ROCKET

Captain YO ACRYLIC/BRASS

DATA COMPONENT

POCKET ROCKET	Captain YO ACR/BRASS
------------------	-------------------------

COMMON:

 $g, \text{m/sec}^2$

9.81

9.81

 J, m

1.0

1.0

VARIABLE:

 M, kg 48.2×10^{-3} 48.5×10^{-3} $I, \text{kg-m}^2$ 7632×10^{-9} 24522×10^{-9} r, m 4×10^{-3} 3×10^{-3}

CALCULATED:

 $k_0 = \left(\frac{I}{M}\right)^{\frac{1}{2}}, \text{m}$ 12.58×10^{-3} 22.49×10^{-3} $KE_s = Mgs, \text{kg-m}^2/\text{sec}^2 \text{ or } \text{N-m}$

0.473

0.476

 $RE_s = KE_s / \left(\left(\frac{r^2}{k_0^2} + 1\right)\right), \text{N-m}$

0.4296

0.4677

 $\omega_s = (2RE_s/I)^{\frac{1}{2}}, \text{rad/sec}$

335.5

195.3

 $\text{RPM}_{\text{EOF}} = \omega_s \left(\frac{30}{\pi}\right), \text{rev/min}$

3204

1865

ENERGY OF ROTATION IMPARTED TO A Yo-Yo DEPENDS ON STRING TO Yo-Yo COUPLING AS THE STRING UNWINDS; TIGHT REWINDING IN THE PRECEDING RETURN IS IMPORTANT.

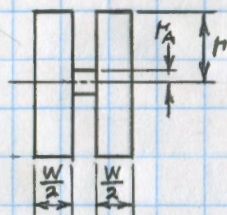
FIXED AXLE Yo-Yos, WITH CAPSTAN AXLE FRICTION, USUALLY INITIATE AND COMPLETE POSITIVE REWINDS, ASSURING WELL-COUPLED UNWINDS.

TRANS AXLE (SLEEVE- OR BALL-BEARING) Yo-Yos, WITH INTENTIONALLY LOW AXLE FRICTION, OFTEN FAIL TO INITIATE A POSITIVE RETURN, DEVELOPING A LOOSE REWIND WITH A RESULTING PARTIAL OR TOTAL SLIP-OUT ON THE NEXT FALL OR THROW. EVEN GIVEN A TIGHT REWIND, AS THE STRING COIL NEARS THE AXLE IN A FALL OR THROW, WEDGED TURNS BETWEEN THE SIDES ARE LOST ALLOWING STRING TO SLIP OUT FROM THERE. ANY SUCH SLIPPAGE MEANS THAT SOME ENERGY OF ROTATION IS LOST TO THE Yo-Yo.

GREASE OR OIL CAN PROVIDE VISCOUS FRICTION IN AXLE BEARINGS TO IMPROVE RELIABLE INITIATION OF REWIND, WHILE CONTROL SYSTEMS ("TURBO" DISKS, FRICTION DISKS, BRAKE PADS, STAKBURSTS, ...) CAN ACT TO IMPROVE COUPLING TO THE Yo-Yo SIDES IN A REWIND.

CALCULATED ENERGY OF ROTATION RE_3 (AND RESULTING ESTIMATES OF ANGULAR VELOCITY, ω_3 AND RPM) MAY NOT BE ACHIEVED IN PRACTICE. ALL SUCH VALUES MUST BE CONSIDERED POTENTIAL RATHER THAN ASSURED. SUCH IS THE NATURE OF MATHEMATICS AND PHYSICS IN THE REAL WORLD.

"HOCKEY PUCK" YO-YOS. IDENTICAL PROFILES YIELD EQUAL YO-YO RADIUS OF GYRATION k_o , ω_s AND RPM_{EOF} . YO-YO WEIGHT M , MOMENT OF INERTIA I , ANGULAR MOMENTUM L , AND ROTATING ENERGY RE_s VARY DIRECTLY WITH THE MATERIAL DENSITY ρ OF THE YO-YO BODY. COMPARING PLASTIC AND ALUMINUM YO-YOS WITH SIMPLE "HOCKEY PUCK" SIDES:



ASSUME:

$$r_A = 4 \times 10^{-3} \text{ m}, \quad r = 1.00 \text{ in}$$

$$W = 0.75 \text{ in}, \quad M = \pi r^2 W \rho \text{ kg}$$

	PLASTIC	ALUMINUM
DENSITY, ρ gm/in ³	≈ 20	≈ 40
WEIGHT, M kg	47×10^{-3}	94×10^{-3}
$I = \frac{1}{2} M r^2$ kg-m ²	15160×10^{-9}	30320×10^{-9}
$k_o = (I/M)^{\frac{1}{2}}$ m	17.96×10^{-3}	
$k_o = [(I/M)^{\frac{1}{2}} / (25.4 \times 10^{-3})]$ in	0.707	
AT STRING END $S = 1.0 \text{ m}$ IN GRAVITY-ONLY FALL:		
$\omega_s = [2gs / (r_A^2 + k_o^2)]^{\frac{1}{2}}$ rad/sec	241	241
$RPM_{EOF} = \omega_s \cdot 9.55$	2302	
$L = I \omega_s$ kg-m ² /sec	3.65×10^{-3}	7.30×10^{-3}
$RE_s = \frac{1}{2} I \omega_s^2$ kg-m ² OR N-m	0.440	0.880

² AMONG YO-YOS OF IDENTICAL PROFILE, THOSE WITH GREATER UNIFORM MATERIAL DENSITY ARE EXPECTED TO REACH EQUAL RPM_{EOF} , BUT SPIN LONGER, GIVEN EQUAL RETARDING TORQUE AT THE AXLE.

1630 - THE NUMERIC YO-YO CONSTANT

POPULAR YO-YOS SELDOM EXCEED AN OUTSIDE DIAMETER OF 2.3 inches; KEY PHYSICAL VARIABLES VERY RARELY EXCEED THE LIMITS LISTED HERE:

KEY VARIABLE	LIMITS
WEIGHT M , kg	$35 \times 10^{-3} < M < 100 \times 10^{-3}$
MOMENT OF INERTIA I , $\text{kg} \cdot \text{m}^2$	$15000 \times 10^{-9} < I < 40000 \times 10^{-9}$
RADIUS OF GYRATION k_o , m	$17.8 \times 10^{-3} < k_o < 25.4 \times 10^{-3}$
" " " k_o , in	$0.70 < k_o < 1.00$
AXLE RADIUS r , m	$3 \times 10^{-3} < r < 5 \times 10^{-3}$
" " " r , in	$0.125 < r < 0.188$

THE "ENERGY APPROACH" IN LEADING TO RPM_{EOF} ALSO LEADS DIRECTLY TO VERTICAL VELOCITY V_3 m/sec FOR THE TETHERED FREE-FALLING YO-YO (SEE MONOGRAPH IV, PAGE 7 FOR AN ALTERNATE DERIVATION FOR V_3). FROM V_3 , A NUMERIC CONSTANT 1630 IS DERIVED; DIVIDED BY k_o IN inches ($0.70 < k_o < 1.00$, ABOVE), 1630 YIELDS RPM_{EOF} WITH LESS THAN $\pm 1.5\%$ ERROR FOR POPULAR YO-YOS AND LESS THAN $\pm 1\%$ FOR MOST MODELS.

ENERGY DEVELOPED IN TETHERED FALL OF 1.0 m:

$$KE_3 = Mgs = \frac{1}{2} MV_3^2 + \frac{1}{2} I \omega_3^2$$

$$\omega_3 = \frac{V_3}{r}; \quad 2gs = V_3^2 + \frac{I V_3^2}{M r^2}$$

$$V_3 = \left[\frac{2gs}{1 + (I/Mr^2)} \right]^{\frac{1}{2}} \text{ m/sec}$$

SUBSTITUTING $I/M = k_o^2$

$$V_s = \left[\frac{295}{1 + (k_o^2/r^2)} \right]^{\frac{1}{2}} = \left(\frac{295r^2}{r^2 + k_o^2} \right)^{\frac{1}{2}} \text{ m/sec}$$

$$\text{RPM}_{\text{EOF}} = \omega_s = \frac{V_s}{r} \cdot \frac{1 \text{ rev}}{2\pi \text{ rad}} \cdot \frac{60 \text{ sec}}{\text{min}} = \frac{30V_s}{\pi r} \text{ rev/min}$$

$$\begin{aligned} \text{RPM}_{\text{EOF}} &= \frac{30}{\pi r} \left(\frac{295r^2}{r^2 + k_o^2} \right)^{\frac{1}{2}} \\ &= \frac{30}{\pi} \left(\frac{2 \cdot 9.81 \cdot 1.0}{r^2 + k_o^2} \right)^{\frac{1}{2}} \end{aligned}$$

FOR r AND k_o IN METERS: $\text{RPM}_{\text{EOF}} = \frac{42.3}{(r^2 + k_o^2)^{\frac{1}{2}}}$

AXLE RADIUS r IS SMALL COMPARED TO RADIUS OF GYRATION k_o ; WITH SOME LOSS OF ACCURACY r MAY BE ELIMINATED FOR A NEAT SIMPLIFICATION WHERE:

$$\text{RPM}_{\text{EOF}} = \frac{42.3}{k_o}$$

THE FACING CHART ANALYZES ERRORS INTRODUCED IN APPLYING THIS SIMPLIFICATION FOR THE NEEDED RANGES OF AXLE RADIUS AND OF RADIUS OF GYRATION. THE CHART INDICATES THAT $42.3/k_o$ REPORTS HIGH VALUES (AS EXPECTED) OF RPM_{EOF} OVER THE FULL RANGES OF r AND k_o ; ERRORS IN THIS CASE RANGE FROM +0.67% TO 3.45%. ADJUSTING 42.3 TO THE LOWER VALUE 41.4 REDUCES THE ERROR LIMITS TO A MORE BALANCED RANGE FROM -1.45% TO +1.66%. RPM_{EOF} ERROR USING 41.4 FOR MOST POPULAR YO-YO MODELS WILL BE LESS THAN 1%. SEE THE RPM_{EOF} CHART IN THE "YO-YO LIMITS" SECTION, PAGE 16.

THE CONSTANT 42.3 AND ITS ALTERNATIVE 41.4 REQUIRE THAT r AND k_0 BE EXPRESSED IN METERS. FOR r AND k_0 IN INCHES, AND FOR BEST ACCURACY, DIVIDING 42.3 BY 25.4×10^{-3} M/IN YIELDS:

$$RPM_{EOF} = \frac{1665}{(r^2 + k_0^2)^{\frac{1}{2}}}$$

FACTORS		RPM _{EOF}				
r	k_0	42.3	42.3	ERR.	41.4	ERR.
		$(r^2 + k_0^2)^{\frac{1}{2}}$	k_0	%	k_0	%
3×10^{-3}	17.8×10^{-3}	2343	2376	+1.41	2326	-0.73
	25.4×10^{-3}	1654	1665	+0.67	1630	-1.45
4×10^{-3}	17.8×10^{-3}	2319	2376	+2.46	2326	+0.30
	25.4×10^{-3}	1645	1665	+1.22	1630	-0.91
5×10^{-3}	17.8×10^{-3}	2288	2376	+3.45	2326	+1.66
	25.4×10^{-3}	1634	1665	+1.90	1630	-0.24

SIMILARLY WITH 41.4, FOR MORE THAN ADEQUATE ACCURACY AND SIMPLICITY:

$$RPM_{EOF} = \frac{1630}{k_0}$$

THE NUMERIC YO-YO CONSTANT 1630 DIVIDED BY A YO-YO RADIUS OF GYRATION IN INCHES YIELDS RPM_{EOF} FOR THE YO-YO, UNWINDING ITS TETHER IN A 110 METER FALL UNDER THE INFLUENCE OF GRAVITY ALONE.

RPM_{Eof} + A GRAPHIC APPROACH

THE KEY VARIABLES: WEIGHT M , MOMENT OF INERTIA I , AND RADIUS OF GYRATION k_o CAN BE USED IN A GRAPH RELATING THEM TO RPM_{Eof}. USING THE MID-VALUE $r = 0.1575 \text{ in } (4 \times 10^{-3} \text{ m})$ AND THE MORE ACCURATE $\text{RPM}_{Eof} = 1665 / (r^2 + k_o^2)^{\frac{1}{2}}$ ARE BEST CHOICES TO CONSTRUCT THE GRAPH.

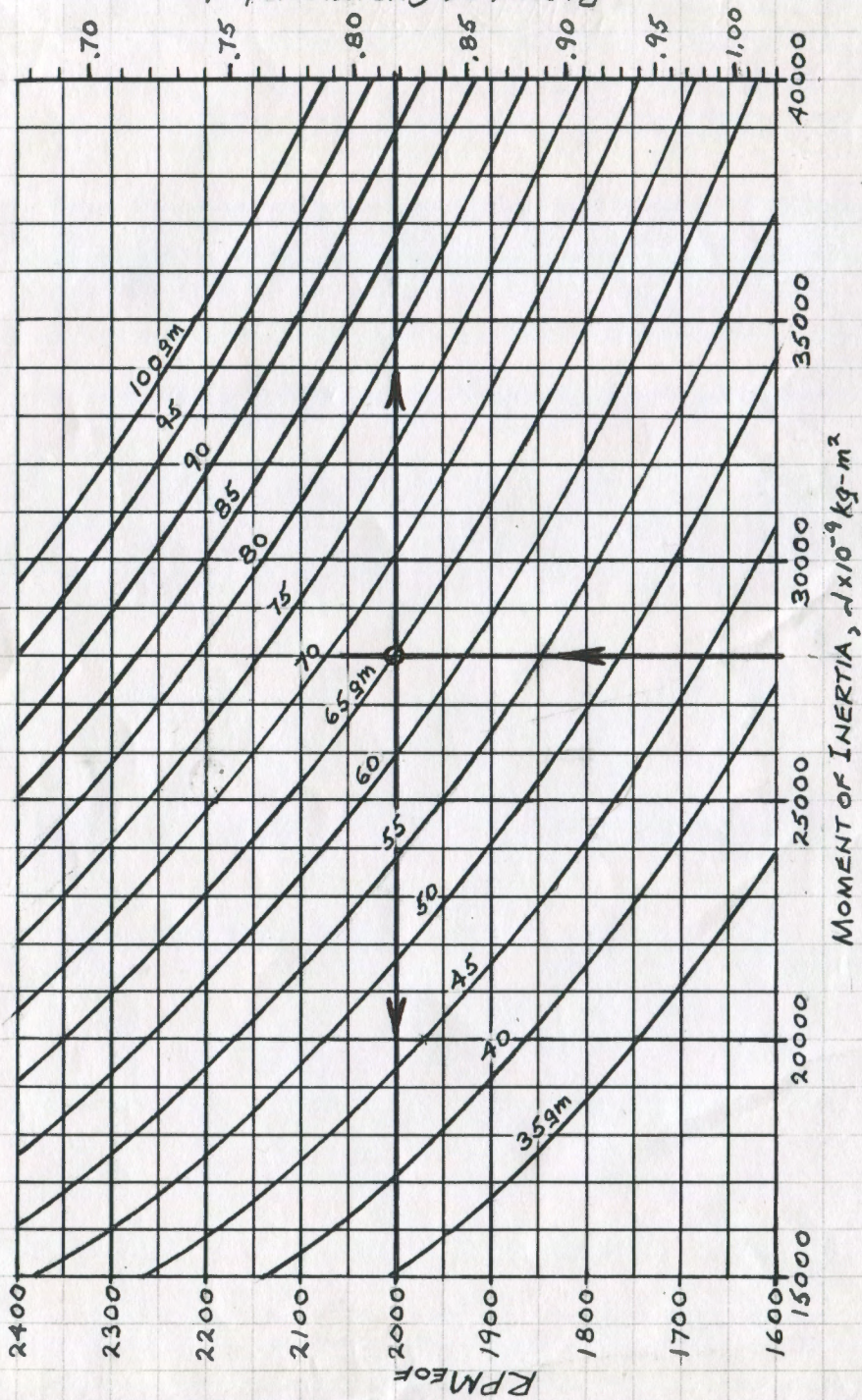
THE FACING CHART IS ASSEMBLED DEVELOPING AT LEAST THREE RPM_{Eof}- k_o PAIRS AT EACH $5 \times 10^{-3} \text{ kg } M$ INCREMENT FOR THE $5000 \times 10^{-9} \text{ kg-m}^2$ I INCREMENTS. FOR EACH SUCH PAIR, $k_o = (I/M)^{\frac{1}{2}} / 25.4 \times 10^{-3}$ IS CALCULATED, THEN RPM_{Eof} IS CALCULATED USING r AND k_o , EACH IN INCHES. THE CHART IS RESTRICTED TO THE $0.70 < k_o, \text{ in } < 1.00$ LIMITS; AS A RESULT, RPM_{Eof} FALLS WITHIN THE LIMITS $1600 < \text{RPM}_{Eof} < 2400$.

ON A MOMENT OF INERTIA VS RPM_{Eof} GRID, THE POINTS FOR EACH WEIGHT INCREMENT ARE PLOTTED AND JOINED SMOOTHLY USING DRAFTING (FRENCH) CURVES TO FORM THE FAMILY $35 < M, \text{ gm } < 100$ OF Y_o-Y_o WEIGHT CURVES.

FOLLOWING THE CHART, THE GRAPH IS PRESENTED WITH SAMPLE VALUES FOR A FICTIONAL Y_o-Y_o WHERE $I = 28000 \times 10^{-9} \text{ kg-m}^2$, $M = 65 \times 10^{-3} \text{ kg}$, AND $\text{RPM}_{Eof} = 2000$. ON THE SCALE AT THE RIGHT, AN ESTIMATED $k_o \approx 0.82 \text{ in}$ CAN BE READ.

USING $\text{RPM}_{Eof} = 1630 / k_o$, $k_o \approx 0.82 \text{ in}$ YIELDS 1988, IN ERROR 0.6%; A MORE ACCURATE CALCULATED $k_o = 0.817 \text{ in}$ YIELDS 1995, IN ERROR 0.3%. GRAPH AND NUMERIC Y_o-Y_o CONSTANT CORRELATE WELL.

$M \times 10^{-3} \text{ kg}$	$k_o \neq \text{RPM}_{\text{EOF}}$					
	MOMENT OF INERTIA, $J \times 10^{-9} \text{ kg-m}^2$					
	15000	20000	25000	30000	35000	40000
100	$\text{RPM}_{\text{EOF}} = \frac{42.3}{(r^2 + k_o^2)^{\frac{1}{2}}}$ <p>WHERE:</p> $r = 4 \times 10^{-3} \text{ m}$ $k_o = \frac{(J \times 10^{-9} / M \times 10^{-3})^{\frac{1}{2}}}{25.4 \times 10^{-3}} \text{ in}$			2380	2211	2074
95				0.682	0.737	0.787
				2322	2157	2023
90				0.700	0.756	0.808
				2263	2102	1971
85	0.719	0.776	0.830			
	2202	2045	1918			
	0.740	0.799	0.854			
	80	$\text{RPM}_{\text{EOF}} \rightarrow$	2334	2139	1986	1862
$k_o, \text{ in} \rightarrow$		0.696	0.762	0.823	0.880	
75		2263	2074	1925	1805	
		0.719	0.787	0.850	0.909	
70		2190	2006	1862	1745	
		0.744	0.815	0.880	0.941	
65		2351	2113	1936	1796	1683
		0.691	0.772	0.846	0.914	0.977
60		2263	2034	1862	1728	1619
		0.719	0.804	0.880	0.951	1.017
55		2171	1950	1785	1656	
		0.751	0.839	0.919	0.993	
50		2380	2074	1862	1704	
		0.682	0.787	0.880	0.964	
45		2263	1971	1769	1614	
		0.719	0.830	0.928	1.016	
40		2189	1862	1671		
		0.762	0.880	0.984		
35		2006	1745			
		0.815	0.941			



RPM_{EOF} - SAMPLE CALCULATION

$$\omega_s = \text{RPM}_{\text{EOF}} = \frac{1665}{(r^2 + k_o^2)^{\frac{1}{2}}} \text{ rev/min}$$

$$\text{LET: } r = 4 \times 10^{-3} \text{ m} = 0.1575 \text{ in}; r^2 = 0.0248 \text{ in}^2$$

$$k_o = 21.3 \times 10^{-3} \text{ m} = 0.84 \text{ in}; k_o^2 = 0.7056 \text{ in}^2$$

$$\text{THEN: } \text{RPM}_{\text{EOF}} = \frac{1665}{(0.7304)^{\frac{1}{2}}} = \frac{1665}{0.855}$$

$$\text{RPM}_{\text{EOF}} = \oplus 1948 \text{ rev/min (SEE BELOW)}$$

RPM_{EOF} FOR $0.68 < k_o < 1.03$ IN 0.01 in INCREMENTS

IS CHARTED HERE FOR THE k_o

X SCALE DATA TO COMPLETE THE

RPM_{EOF} - r - M - k_o GRAPH.

k_o	RPM _{EOF}
0.68	2385
0.69	2353
0.70	2321
0.71	2289
0.72	2259
0.73	2230
0.74	2201
0.75	2173
0.76	2145
0.77	2118
0.78	2092
0.79	2067

0.80	2042
0.81	2018
0.82	1994
0.83	1971
0.84	\oplus 1948
0.85	1926
0.86	1904
0.87	1883
0.88	1862
0.89	1842

0.90	1822
0.91	1803
0.92	1784
0.93	1765
0.94	1747
0.95	1729
0.96	1711
0.97	1694
0.98	1677
0.99	1661
1.00	1645
1.01	1629
1.02	1613
1.03	1598

NOTE: IN *"EXCEL", RPM_{EOF} =
 $\text{ROUND}((1665/(r^2 + k_o^2))^{\frac{1}{2}}, 0)$

YO-YO LIMITS - CHARTED AND GRAPHED

THE $RPM_{EOF} = 1630/k_0$ DATA WERE DEVELOPED FOR YO-YOS 1 THROUGH 13 USING ACTUAL MODELS, WITH METHODS GIVEN IN MONOGRAPHS I AND II. PRODUCTION MODELS ARE SUBJECT TO CHANGE.

THE "FREEHAND"-(1) IS LISTED WITH OPTIONAL WEIGHT RINGS; "FREEHAND"-(2) IS THE BASIC MODEL. "ECLIPSE"-(1) IS THE BASIC MODEL; MODELS -(2, 3, AND 4) ARE LISTED WITH RING COMBINATIONS OF INCREASING WEIGHT.

THE "3B2.2" IS LISTED WITH OPTIONAL WEIGHT RINGS MOUNTED.

THE CAPTAIN YO MODEL IS A NON-COMMERCIAL ONE-OF-A-KIND, BUILT TO TEST EXTREME (RIM-) WEIGHT DISTRIBUTION, USING BRASS RIMS ON ~~MINI~~ LIGHT (THIN) CROSS-SECTION ACRYLIC HALVES.

THE BÜRGER (PLAYABLE) AT 45gm AND THE "LONG SPIN" AT 60gm EVOLVE FROM BÜRGER'S 1984 ARTICLE ANALYZING A SAMPLE YO-YO WITH WEIGHT AND MOMENT OF INERTIA NOT GIVEN, BUT WITH RADIUS OF GYRATION "OF ABOUT 2.5cm" (≈ 0.984 in). WHEN MONOGRAPH IV WAS WRITTEN (2001), THE "PLAYABLE YO-YO" WAS ASSIGNED $M = 45.0$ gm AND, WITH ITS QUOTED $k_0 = 2.5$ cm, THE CALCULATED $I = 28125 \times 10^{-9} \text{ kg-m}^2$.

AN EARLY DUNCAN "LONG SPIN" (WHEELS), MUCH LATER ACQUIRED, WAS WEIGHED AT 60gm AND I THEN RECALCULATED TO $37500 \times 10^{-9} \text{ kg-m}^2$ WITH $k_0 = 2.5$ cm. BÜRGER ALSO ESTIMATED AN RPM_{EOF} OF 1800; FOR $k_0 \approx 1630/1800 \approx 0.91$ in AND $I \approx 32000 \text{ kg-m}^2$.

$$RPM_{Eof} = \frac{1630}{k_0}$$

No.	Yo-Yo	J $\times 10^{-9} \text{ kg-m}^2$	M $\times 10^{-3} \text{ kg}$	r $\times 10^{-3} \text{ m}$	V_3 m/sec	k_0 in	$\frac{1630}{RPM_{Eof}}$ $\frac{1630}{k_0}$	% ERR.
1	BC/TK "POCKET ROCKET"	7632	48.2	4.0	1.342	0.495	3204	3293 +2.8
2	BC/TK "NO LIVE 3-IN-1"	17803	54.4	3.2	0.772	0.712	2304	2289 -0.7
3	DUNCAN "FREEHAND" -(1)	24506	65.2	5.0	1.106	0.763	2112	2136 +1.1
4	" -(2)	21657	57.2	5.0	1.102	0.766	2105	2128 +1.1
5	RUSSELL "SUPER"	17300	45.0	4.0*	0.885	0.772	2113	2111 -0.1
6	SPINTASTICS "ECLIPSE" -(1)	25223	65.4	4.8	1.052	0.773	2093	2109 +0.8
7	" (2)	28511	72.8	4.8	1.044	0.779	2077	2092 +0.7
8	CUSTOM "CHAIN REACTOR"	19717	50.2	4.8	1.043	0.780	2075	2090 +0.7
9	SPINTASTICS "ECLIPSE" -(3)	32914	79.8	4.8	1.019	0.800	2027	2038 +0.5
10	" -(4)	36202	87.2	4.8	1.016	0.802	2021	2032 +0.5
11	" "TECHNIC"	22391	50.0	3.4	0.703	0.833	1974	1957 -0.9
12	BC/TK "5B2.2"	30400	60.8	4.0*	0.780	0.880	1862	1852 -0.5
13	Captain YO (ACRYLIC/BRASS)	24522	48.5	3.0	0.586	0.885	1865	1842 -1.2
14	BÜRGER (PLAYABLE)	28125	45.0	3.0	0.528	0.984	1681	1657 -1.4
15	DUNCAN "LONG SPIN"	37500	60.0	3.0	0.528	0.984	1681	1657 -1.4

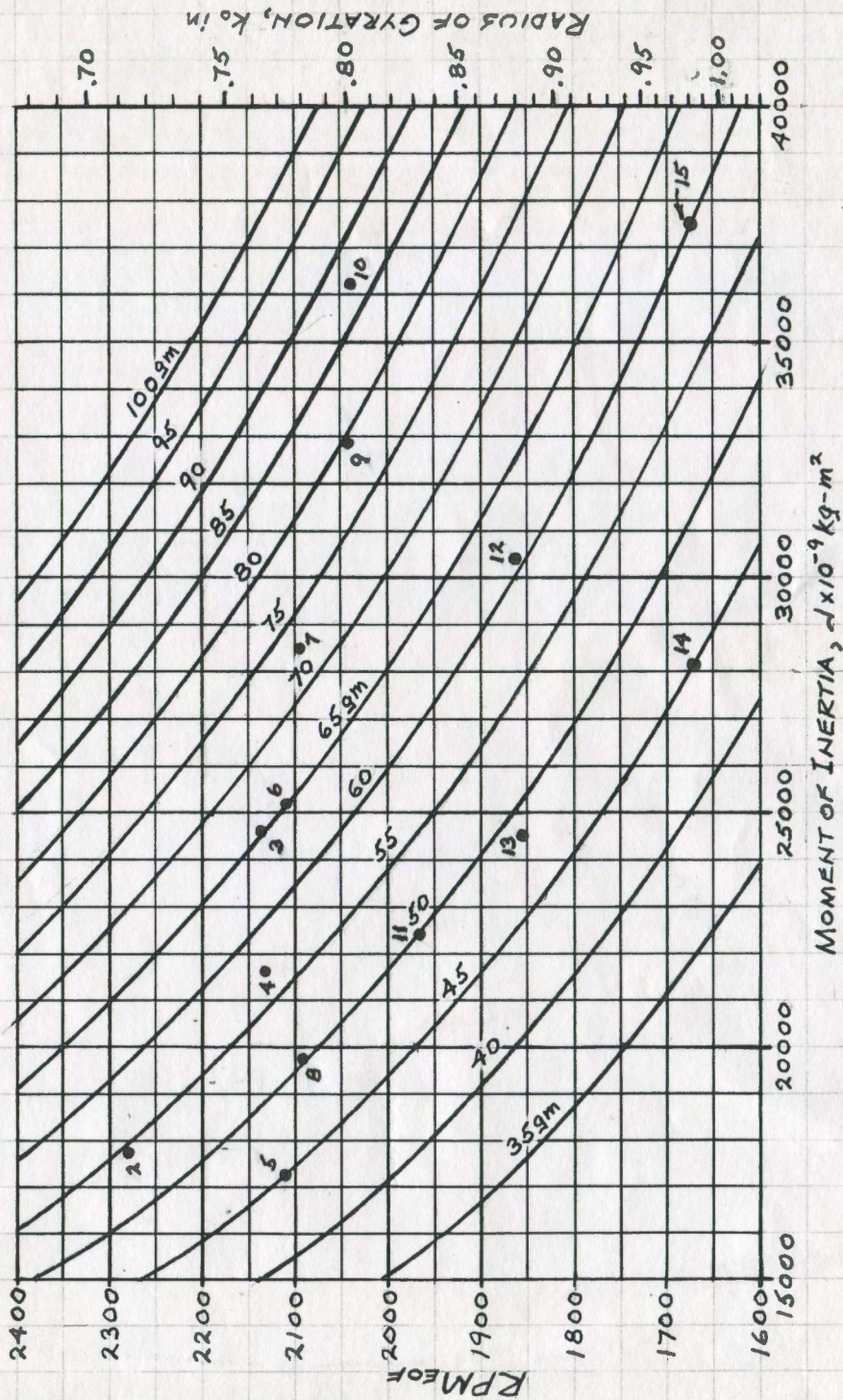
THE CHART LISTING IS IN ORDER OF INCREASING k_0 . TRANSITION TO LOWER AXLE RADIUS r FROM Y_0-Y_0s 4 TO 5 AND FROM 12 TO 13 EACH SHOW A SLIGHT INCREASE IN $RPM_{EOF} = \omega_s$ FROM $V_s = 42.3/(r^2 + k_0^2)^{1/2}$; (r AND k_0 IN METERS). FOR $RPM_{EOF} = 1630/k_0$, r BEING IGNORED, RPM_{EOF} STEADILY DECREASES; RELATIVE ERROR % FOR EACH LISTING IS GIVEN.

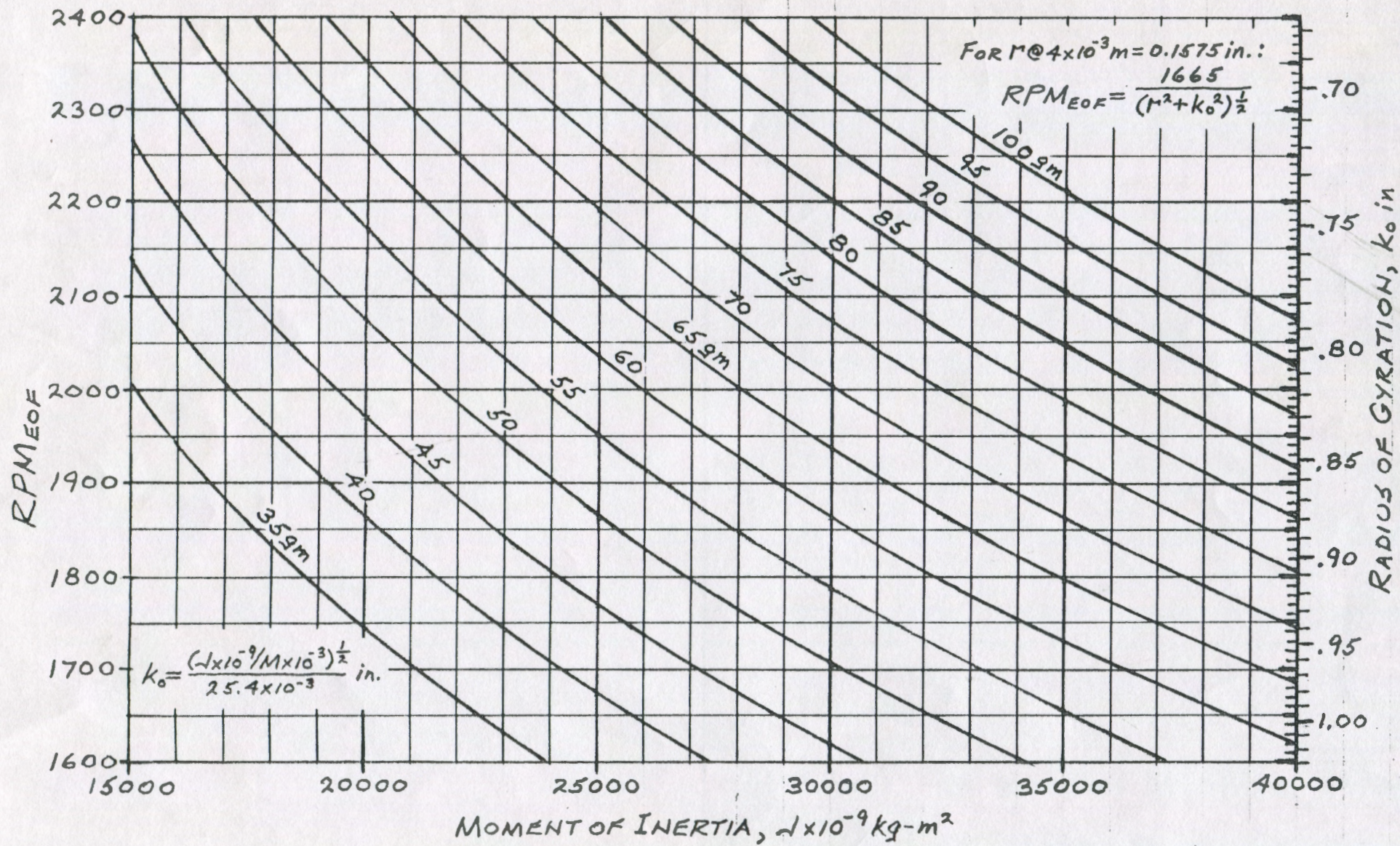
VALUES J AND M FOR EACH VARIATION ARE EASILY USED TO PLOT THE RESPECTIVE LOCATIONS ON THE GRAPH EARLIER DEVELOPED.

THE "POCKET ROCKET" WOULD LIE ON A WEIGHT LINE $M = 48.2 \text{ gm}$. BUT, AT $J = 7632 \times 10^{-9} \text{ kg-m}^2$, WELL OUTSIDE THE UPPER LEFT CORNER OF THE GRAPH, AT $RPM_{EOF} \approx 3200$, MAKING IT (BY FAR!) THE FASTEST SPINNER IN THE GROUP. THE CAPTAIN Y_0 MODEL ACHIEVES ONLY $RPM_{EOF} \approx 1860$. THESE TWO Y_0-Y_0s ARE AT THE LIMITS OF THE 13 TESTED VARIATIONS.

THE BÜRGER (PLAYABLE) AND "LONG SPIN" LISTINGS INCLUDE ESTIMATED DATA RATHER THAN DATA MEASURED OR TESTED. $RPM \approx 1800$ ESTIMATED BY BÜRGER AND $M = 60 \text{ gm}$ (MEASURED) BRING THE TWO LISTINGS FOR THE EARLY "LONG SPIN" WHEELS TOGETHER AT $k_0 = 0.91 \text{ in.}$ AND $J \approx 32000 \text{ kg-m}^2$. CHECK THAT POINT OUT ON THE GRAPH.

Y_0-Y_0s marginally rim-weighted (or not at all so) TEND TOWARD THE UPPER LEFT AREA OF THE GRAPH; SIGNIFICANTLY RIM-WEIGHTED Y_0-Y_0s TEND TOWARD THE LOWER RIGHT AREA.





WEIGHT DISTRIBUTION. RIM WEIGHTING IN A YO-YO DESIGN CAN BE DEFINED AS A RATIO OF RADIUS OF GYRATION k_o TO OUTSIDE RADIUS OR OF THE YO-YO, EXPRESSED AS A PERCENTAGE:

$$RW\% = \frac{k_o}{OR} \cdot 100\%$$

THE FACING CHART LISTS k_o IN INCREASING ORDER. DIFFERENCES IN OUTSIDE RADIUS OR AFFECT THE ORDER OF $RW\%$. DIFFERENCES IN WEIGHT M AND AXLE RADIUS r (NEITHER LISTED HERE, SEE PAGE 17), AFFECT THE ORDER OF ANGULAR MOMENT OF INERTIA I AND RPM_{EOF} (WITH ω_s), RESPECTIVELY. THE GENERAL TREND OF I IS INCREASING WHILE THE GENERAL TREND OF RPM_{EOF} AND ω_s IS DECREASING. ANGULAR MOMENTUM L_s VARIES DIRECTLY WITH I AND DIRECTLY WITH ω_s ; ENERGY OF ROTATION RE_s VARIES DIRECTLY WITH I AND DIRECTLY WITH ω_s^2 .

ANGULAR MOMENTUM L_s $\text{kg} \cdot \text{m}^2/\text{sec}$ IS OF SPECIAL INTEREST IN EVALUATING YO-YO SPIN DURATION. IN THIS LISTING FOR GRAVITY-ONLY FALLS, ANGULAR MOMENTUM L_s FOR THE "SB2.2" IS MORE THAN TWICE THAT FOR THE "POCKET ROCKET"; GIVEN BEARINGS OF EQUAL QUALITY, CONDITION AND RETARDING TORQUE, THE "SB2.2" SPIN DURATION SHOULD BE MORE THAN TWICE THAT OF THE "POCKET ROCKET" AFTER UNAIDED 1.0m FALL - DESPITE $RPM_{EOF} \approx 3200$ FOR THE "POCKET ROCKET" AND $RPM_{EOF} \approx 2000$ FOR THE "SB2.2". FOR MUCH MORE DETAIL ON THIS GENERAL SUBJECT, SEE "SLEEPER" SPIN DURATION, PAGE 30.

RIM WEIGHTING				ANGULAR MOMENTUM AND ENERGY OF ROTATION					
No.	Yo-Yo	K _o	OR	RW	I	RPM _{EOF}	ω _s	L _s = Iω _s	RE = $\frac{1}{2}$ Iω _s ²
		in	in	%	10 ⁻⁹ kg-m ²	From V _s	rad/sec	kg-m ² /sec	N-m
1	"POCKET ROCKET"	0.495	0.750	66.0	7632	3204	335.5	2.56x10 ³	0.430
2	"NO-LIVE 3-IN-1"	0.712	1.130	63.0	17803	2304	241.2	4.29x10 ³	0.518
3	"FREEHAND"-(1)	0.763	1.120	68.1	24506	2112	221.1	5.42x10 ³	0.599
4	"-(2)	0.766	1.120	68.4	21657	2105	220.4	4.77x10 ³	0.526
5	"SUPER"	0.772	1.120	68.9	17300	2113	221.2	3.83x10 ³	0.423
6	"ECLIPSE"-(1)	0.773	1.120	69.0	25223	2093	219.1	5.53x10 ³	0.605
7	"-(2)	0.779	1.120	69.6	28511	2077	217.5	6.20x10 ³	0.674
8	"CHAIN REACTOR"	0.780	0.990	78.8	19717	2075	217.3	4.28x10 ³	0.466
9	"ECLIPSE"-(3)	0.800	1.120	71.4	32914	2027	212.2	6.98x10 ³	0.741
10	"-(4)	0.802	1.120	71.6	36202	2021	211.6	7.66x10 ³	0.810
11	"TECHNIC"	0.833	1.135	73.4	22391	1974	206.7	4.63x10 ³	0.478
12	"SB2.2"	0.880	1.115	78.9	30400	1862	195.0	5.93x10 ³	0.578
13	Captain Yo (A/B)	0.885	1.080	81.9	24522	1865	195.3	4.79x10 ³	0.468
14	(PLAYABLE)	0.984	1.125	87.5	28125	1681	176.0	4.95x10 ³	0.436
15	"LONG SPIN"	0.984	1.125	87.5	37500	1681	176.0	6.60x10 ³	0.581

A "SLEEPER" ANALYSIS

ENERGY BALANCE. IN AN ISOLATED SYSTEM, THE CONSERVATION OF ENERGY PRINCIPLE ALLOWS THAT ENERGY MAY BE CONVERTED FROM ONE FORM TO ANOTHER SO LONG AS THE TOTAL SYSTEM ENERGY REMAINS CONSTANT; ENERGY WITHIN THE SYSTEM IS NEITHER CREATED NOR DESTROYED.

IN A VERTICAL "SLEEPER", THE YO-YO AND STRING CAN BE CONSIDERED AN ISOLATED MECHANICAL SYSTEM FROM THE INSTANT THE YO-YO LEAVES THE HAND TO THE INSTANT IT REACHES THE STRING END. IN THE "SLEEPER", THE INITIAL ENERGY OF ROTATION CLEARLY CAN'T EXCEED THE SUM OF VERTICAL ENERGY OF TRANSLATION GIVEN THE YO-YO FROM TWO SOURCES: VERTICAL VELOCITY AT POINT OF RELEASE, AND GRAVITY ACCELERATION FOR THE EFFECTIVE LENGTH OF THE STRING.

THE YO-YO EFFICIENTLY CONVERTS LINEAR TO ROTARY MOTION, BUT IT REACHES THE STRING END WITH RESIDUAL VERTICAL VELOCITY; THUS, SOME FRACTION OF VERTICAL THROW AND GRAVITY ENERGY TOTAL IS NOT CONVERTED TO ENERGY OF YO-YO ROTATION. THE "SLEEPER" INITIAL ROTATIONAL ENERGY CANNOT EQUAL (MUCH LESS "EXCEED") THE TOTAL ENERGY PROVIDED.

THE INITIAL (AND MAXIMUM) "SLEEPER" ANGULAR VELOCITY OCCURS ONLY AT THE INSTANT THE YO-YO REACHES THE STRING END; IT CAN BE CALCULATED IN AN ENERGY BALANCE ANALYSIS.

THE "SLEEPER" EQUATIONS. SEVERAL DEFINITIONS ARE NEEDED.

KINETIC ENERGY FACTORS:

KE_T = TOTAL KINETIC ENERGY APPLIED

TE_R = ENERGY OF TRANSLATION AT RELEASE

E_g = GRAVITY-INDUCED ENERGY TO END OF STRING

KE_s = TOTAL KINETIC ENERGY AT END OF STRING

TE_s = ENERGY OF TRANSLATION AT END OF STRING

RE_s = ENERGY OF ROTATION AT END OF STRING

VARIABLES AND *CONSTANTS:

M = YO-YO WEIGHT, $kg \times 10^{-3}$

V_R = VERTICAL VELOCITY AT RELEASE, m/sec

* g = GRAVITY-INDUCED ACCELERATION, 9.81 m/sec^2

* s = LENGTH OF FALL AFTER RELEASE, 1.0 m

V_s = VERTICAL VELOCITY AT $s = 1.0 \text{ m}$

I = ANGULAR MOMENT OF INERTIA, $kg \cdot m^2 \times 10^{-9}$

ω_s = ANGULAR VELOCITY AT $s = 1.0 \text{ m}$, rad/sec

* r = YO-YO AXLE RADIUS, $4 \times 10^{-3} \text{ m}$

k_o = RADIUS OF GYRATION, $m \times 10^{-3}$

BALANCING ENERGY APPLIED AND CONVERTED:

$$KE_T = KE_s$$

$$TE_R + E_g = TE_s + RE_s$$

$$\frac{1}{2} M V_R^2 + M g s = \frac{1}{2} M V_s^2 + \frac{1}{2} I \omega_s^2$$

At $s = 1.0 \text{ m}$; $V_s = r\omega_s$, $V_s^2 = r^2\omega_s^2$:

$$\frac{1}{2}MV_R^2 + Mgs = \frac{1}{2}Mr^2\omega_s^2 + \frac{1}{2}I\omega_s^2$$

$$M(V_R^2 + 2gs) = \omega_s^2(Mr^2 + I)$$

$$\omega_s = \left(\frac{V_R^2 + 2gs}{r^2 + \frac{I}{M}} \right)^{\frac{1}{2}}$$

$$I = Mk_o^2; \frac{I}{M} = k_o^2$$

$$\omega_s = \left(\frac{V_R^2 + 2gs}{r^2 + k_o^2} \right)^{\frac{1}{2}} \text{ rad/sec}$$

FOR CONVENIENCE, INITIAL "SLEEPER" ANGULAR VELOCITY ω_s CAN BE EXPRESSED AS RPM_s AT $s = 1.0 \text{ m}$ USING $1 \text{ rad/sec} = 9.55 \text{ rev/min}$; V_R AND $2gs$ MAY BE USED IN mi/hr UNITS WHERE $1 \text{ mi/hr} = 0.447 \text{ m/sec}$; r AND k_o MAY BE USED IN IN UNITS WHERE $1 \text{ in} = 25.4 \times 10^{-3} \text{ m}$:

$$2gs = 19.62 \text{ m}^2/\text{sec}^2 \cdot \frac{1.0^2 \text{ mi}^2/\text{hr}^2}{0.447^2 \text{ m}^2/\text{sec}^2} = 98.19 \text{ mi}^2/\text{hr}^2$$

$$\omega_s \text{ rad/sec} = \text{RPM}_s = \frac{(V_R^2 + 2gs)^{\frac{1}{2}} \cdot 0.447}{(r^2 + k_o^2)^{\frac{1}{2}} \cdot 25.4 \times 10^{-3}} \cdot 9.55$$

ACCUMULATING CONVERSION FACTORS:

$$\text{RPM}_s = 168.06 \left(\frac{V_R^2 + 98.19}{r^2 + k_o^2} \right)^{\frac{1}{2}}$$

RPM_s SOLVED FOR V_R :

$$V_R = \left[\frac{\text{RPM}_s^2 (r^2 + k_o^2)}{168.06^2} - 98.19 \right]^{\frac{1}{2}} \text{ mi/hr}$$

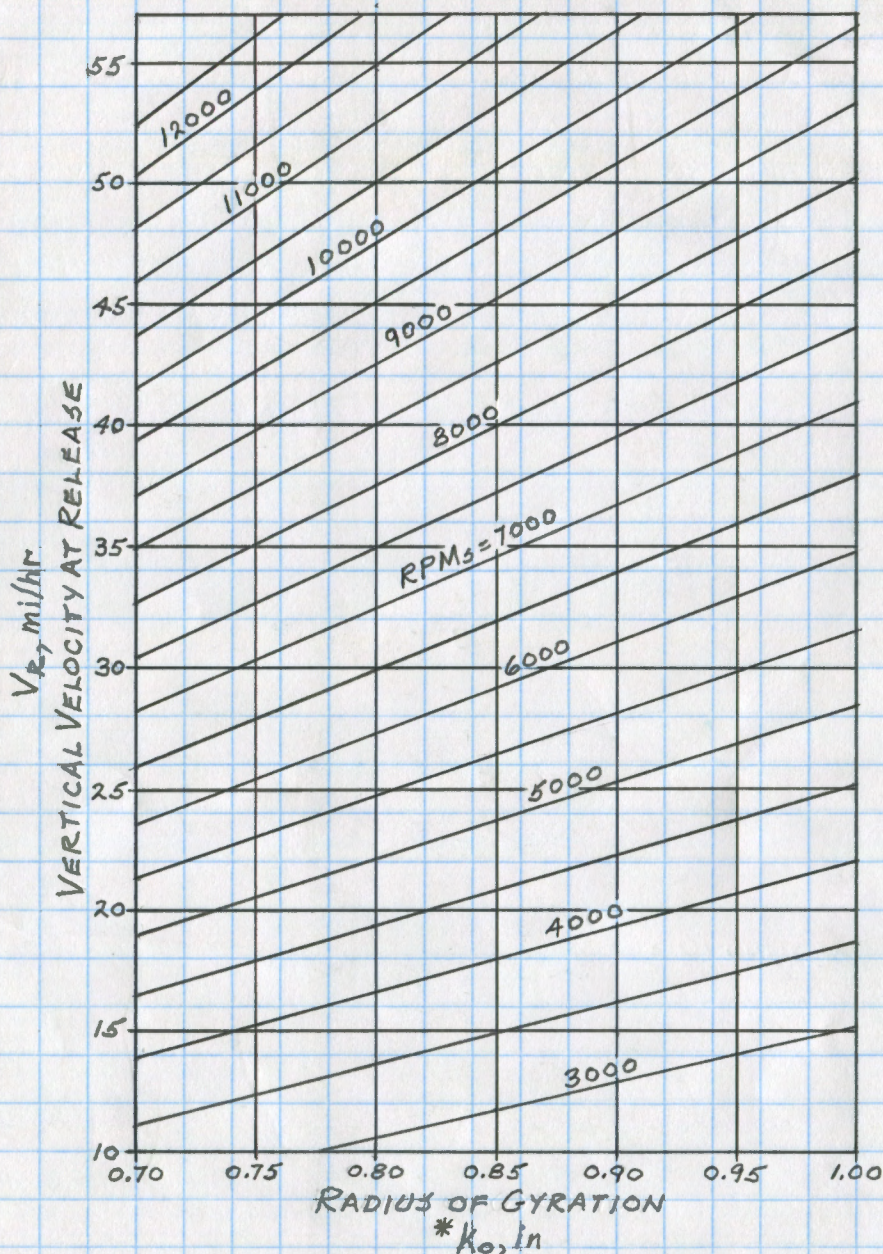
THE "SLEEPER" EQUATIONS ARE GRAPHED ON THE FOLLOWING PAGES; FOR ANY CONSTANT V_R , RPM_s SENSITIVITY TO k_o IS SHOWN ON THE FIRST GRAPH; FOR ANY CONSTANT RPM_s , V_R SENSITIVITY TO k_o IS SHOWN ON THE SECOND. ANY POINT ON EITHER GRAPH HAS ITS EQUIVALENT ON THE OTHER.

SIMPLIFIED VERSIONS OF THE EQUATIONS YIELD RESULTS WITH LESS THAN 2% ERROR AGAINST THE DERIVED ORIGINALS. IN SIMPLIFYING, AXLE RAD-103 R IS ELIMINATED AND 98.19 IS ROUNDED TO 100; ERRORS ARE THEN MINIMIZED BY REDUCING 168.06 TO 163. WITH V_R mi/hr AND k_o in:

$$RPM_s = \frac{163(V_R^2 + 100)^{\frac{1}{2}}}{k_o}$$

$$V_R = \left[\left(\frac{RPM_s \cdot k_o}{163} \right)^2 - 100 \right]^{\frac{1}{2}} \text{ mi/hr}$$

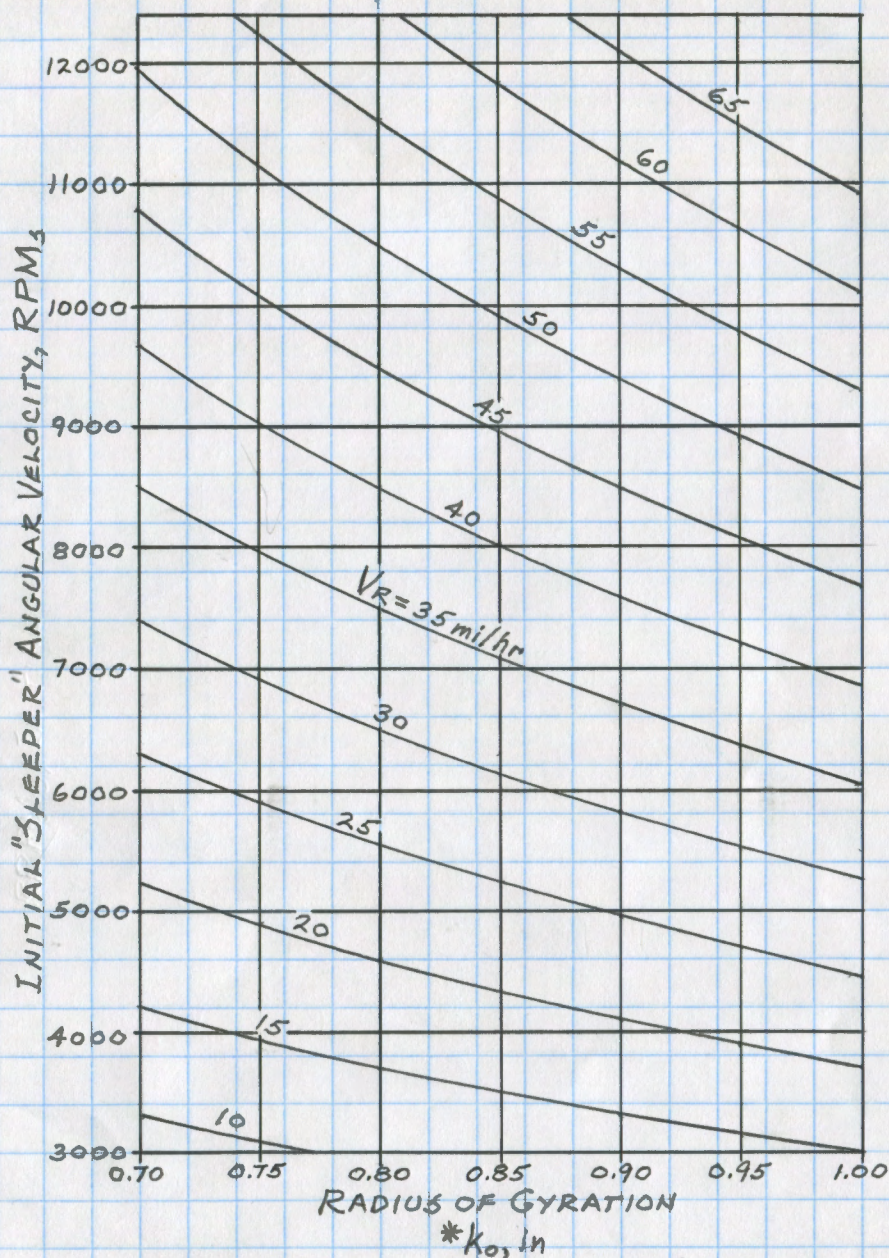
"SLEEPER" V_R IN mi/hr? PROFESSIONAL BASE-BALL PITCHERS COMMONLY THROW "FASTBALLS" AT OR AROUND 100 mi/hr, AND OTHER PITCHES AT MUCH LOWER VELOCITY. RATING YO-YO THROWS IN mi/hr PROVIDES RATIONAL COMPARISON. RECENTLY, A FEW EXPERIENCED PLAYERS, ASKED TO THROW AS THEY WOULD IN NORMAL PLAY, THREW A YO-YO WITH $k_o = 0.83$ in TO TACHOMETER-MEASURED RPM_s VALUES FROM 5800 TO 6700. BY CALCULATION OR GRAPH, $V_R \approx 30$ mi/hr OR 28 TO 32 mi/hr FOR THE GROUP IN NORMAL RATHER THAN EXTRAORDINARY PLAYING EFFORT. WHAT "SLEEPER" RPM_s MIGHT BE REACHED WITH $V_R = 100$ mi/hr WITH THIS YO-YO?



FOR r , K_0 , AND V_R :

* $M = 0.1575 \text{ in}$

$$RPM_s = 168.06 \left(\frac{V_R^2 + 98.19}{r^2 + K_0^2} \right)^{\frac{1}{2}}$$



FOR r , k_0 , AND RPM_3 :

$$* r = 0.15751 m$$

$$V_R = \left[\frac{RPM_3^2 (r^2 + k_0^2)}{168.06^2} - 98.19 \right]^{\frac{1}{2}} \text{ mi/hr}$$

"SLEEPER" SPIN DURATION. YO-YO ANGULAR MOMENTUM AND AXLE BEARING RETARDING TORQUE DETERMINE SPIN DURATION FOR ANY KNOWN YO-YO INITIAL "SLEEPER" ANGULAR VELOCITY. SPIN DURATION FOR TWO YO-YOS OF EQUAL WEIGHT, BUT OF SIGNIFICANTLY DIFFERENT WEIGHT DISTRIBUTION, IS OUTLINED BELOW AND DETAILED IN THE FOLLOWING CHART:

1. USING THE LISTED WEIGHT M AND MOMENT OF INERTIA I , CALCULATE k_o IN FOR EACH YO-YO.

2. ASSUME A COMMON VERTICAL VELOCITY V_k mi/hr AT RELEASE FROM THE HAND.

3. BY CALCULATION, OR FROM EITHER GRAPH, DETERMINE "SLEEPER" INITIAL ANGULAR VELOCITY RPM_s ; CALCULATE $\omega_s = (RPM_s / 9.55) \text{ rad/sec}$.

4. CALCULATE "SLEEPER" INITIAL ANGULAR MOMENTUM $L = I \text{ kg-m}^2 \cdot \omega_s \text{ rad/sec}$. $I \omega_s \text{ kg-m}^2/\text{sec}$ WHEN MULTIPLIED BY sec/sec YIELDS $(\text{kg-m}^2/\text{sec}^2) \cdot \text{sec}$ OR Newton-m-sec ; THEN $L = I \omega_s \text{ N-m-sec}$.

5. RETARDING TORQUE T_B FOR SMALL BALL BEARINGS IS COMMONLY STATED IN gm-cm UNITS. CONVERTED TO Newton-m UNITS:

$$1.0 \text{ gm-cm} \cdot (1.0 \text{ N} / 101.9 \text{ gm}) \cdot (1 \text{ m} / 100 \text{ cm}) = 9.81 \times 10^{-5} \text{ N-m}$$

A REASONABLE RETARDING TORQUE FOR CLEAN, DRY (OR LIGHTLY LUBRICATED), UNSEALED YO-YO BEARINGS IS $T_B = 0.50 \text{ gm-cm}$ OR $4.91 \times 10^{-5} \text{ N-m}$.

6. CALCULATE "SLEEPER" SPIN DURATION D_s min:

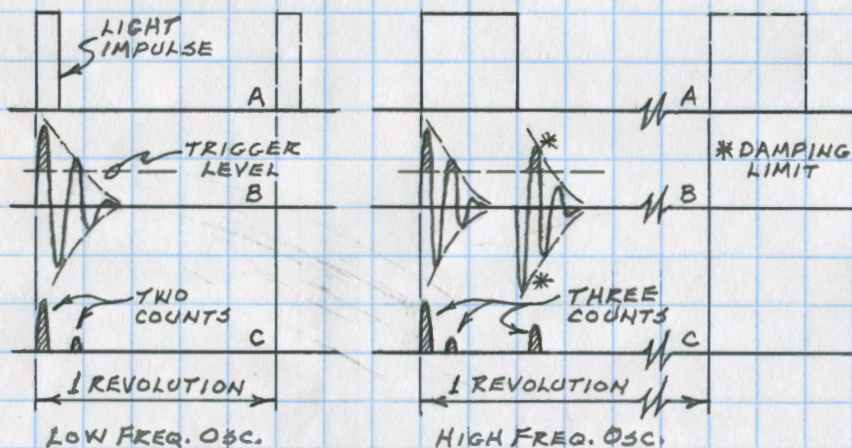
$$D_s = \frac{L \text{ N-m-sec}}{T_B \text{ N-m}} \cdot \frac{1 \text{ min}}{60 \text{ sec}}$$

"SLEEPER" SPIN DURATION	Yo-Yo A	Yo-Yo B
1. WEIGHT, $M \times 10^{-3} \text{ kg}$	75	75
MOMENT OF INERTIA, $I \times 10^{-9} \text{ kg-m}^2$	27200	39200
RADIUS OF GYRATION, $K_o = (I/M)^{1/2} \cdot 25.4 \times 10^{-3} \text{ in}$	0.75	0.90
2. VELOCITY AT RELEASE, $V_R \text{ mi/hr}$	30	30
3. INITIAL ANGULAR VELOCITY, RPM_s	6900	5800
$\omega_s = (\text{RPM}_s / 9.55) \text{ rad/sec}$	723	607
4. INITIAL ANGULAR MOMENTUM, $L = I \omega_s \text{ kg-m}^2/\text{sec OR N-m-s}$	19.7×10^{-3}	23.8×10^{-3}
5. BEARING TORQUE, $T_B \text{ gm-cm}$	0.50	0.50
$T_B \cdot 9.81 \times 10^{-5} \text{ N-m}$	4.91×10^{-5}	4.91×10^{-5}
6. SPIN DURATION, $D_s = (L / 60 T_B) \text{ min}$	6.7	8.1
$D_s, \text{ min: sec}$	6:42	8:06

NOTICE IN THE CHART THAT K_o , L , AND D_s ARE IN THE SAME RATIO ($\approx 1.20:1$) FROM Yo-Yo B TO Yo-Yo A; RPM_s AND ω_s ARE IN THE INVERSE RATIO ($\approx 1:1.20$).




Yo-Yo BEARING RETARDING TORQUE CAN VARY WIDELY, BUT GIVEN EQUAL WEIGHT, RETARDING TORQUE AND VELOCITY OF THROW, BIASING Yo-Yo WEIGHT TOWARD THE RIM (FOR GREATER MOMENT OF INERTIA AND RADIUS OF GYRATION) RESULTS IN GREATER INITIAL ANGULAR MOMENTUM AND SPIN DURATION DESPITE THE LESSER INITIAL "SLEEPER" ANGULAR VELOCITY.

TACHOMETER TRIALS. "NON-CONTACT" TACHOMETERS SENSE LIGHT IMPULSES REFLECTED FROM A PATCH (ABOUT $\frac{3}{8}$ " SQUARE) OF REFLECTIVE TAPE ATTACHED NEAR THE YO-YO RIM. SOME TACHOMETERS EXHIBIT A MULTIPLE "TICKING" FAULT, REGISTERING TWO OR MORE COUNTS FOR EACH YO-YO REVOLUTION. DAMPED OSCILLATION SIGNALS IN PLACE OF CLEAN SINGLE PULSE SIGNALS AT THE TACHOMETER INPUT ARE THE LIKELY CAUSES; TWO POSSIBLE MODES OF MULTIPLE "TICKING" ARE PORTRAYED HERE.



COMBINATIONS OF CIRCUIT STRAY CAPACITANCE AND INDUCTANCE CAN CAUSE OSCILLATION IN THE TACH INPUT SIGNAL; RESISTANCE IN THE CIRCUIT DETERMINES THE DAMPING EFFECT. LOW OSCILLATION FREQUENCY AND POOR DAMPING MAY EXTEND THE OSCILLATION BEYOND THE LIGHT IMPULSE DURATION; HIGHER FREQUENCY AND GOOD DAMPING MAY LIMIT THE OSCILLATION WITHIN THE LIGHT IMPULSE DURATION. IN EACH CASE, TIME LINE A SHOWS THE LIGHT IMPULSE "SEEN" BY THE TACH

PHOTORECEPTOR. TIME LINE B ILLUSTRATES DAMPED OSCILLATION "RINGING" SIGNALS, THE MOST LIKELY CAUSES OF MULTIPLE "TICKING". TIME LINE C SHOWS THE FILTERED (AT TRIGGER LEVEL) SIGNALS AT THE TACHOMETER COUNTER INPUT. IF THE PHOTORECEPTOR IS COUPLED TO THE TACHOMETER FILTER INPUT THROUGH A RESISTANCE-LOADED CAPACITOR, A SECOND OSCILLATION (NEGATIVE-STARTING) CAN OCCUR AT THE LIGHT IMPULSE TRAILING EDGE.

IN THE LOW FREQUENCY CASE, THE TRIGGER LEVEL DETERMINES THAT ONE OR TWO (TWO SHOWN) COUNTS PER REVOLUTION MAY REGISTER; IN THE HIGH FREQUENCY CASE, THE TRIGGER LEVEL DETERMINES THAT ONE OR MORE COUNTS (UP TO FOUR?) PER REVOLUTION MAY REGISTER. IN EITHER CASE, IT MAY BE POSSIBLE TO ELIMINATE THE MULTIPLE "TICKING" BY MOUNTING THE PATCH WITH ITS EDGES AT 45° () RATHER THAN PARALLEL () TO THE RADIUS AT THE RIM. THIS TECHNIQUE ADJUSTS THE LIGHT IMPULSE TO A TRAPEZOIDAL SHAPE () AT TIME LINE A, SOFTENING THE RATE OF RISE (AND FALL) OF THE IMPULSE, DELIVERING CLEANER SIGNALS TO THE COUNTER.

TACHOMETER TESTING A YO-YO SIDE ON A LATHE AT KNOWN RPM MAY REVEAL MULTIPLE "TICKING." IF IT'S PRESENT, BE AWARE IN REAL YO-YO TRIALS.

IN ACTUAL USE, THE TACHOMETER MUST BE AIMED PERPENDICULAR TO, AND DIRECTLY AT, THE PASSING REFLECTIVE PATCH IN THE EARLIEST INSTANT AS THE YO-YO REACHES THE END OF THE STRING TO REGISTER THE INITIAL (MAXIMUM) "SLEEPER" RPM.

STRING COUPLING. IN A FIXED WOOD AXLE YO-YO WITH A SNUG STRING LOOP AT THE AXLE, THE FIRST FEW TURNS IN A "SLEEPER" RETURN WEDGE FIRMLY IN THE GAP, ASSURING TIGHT REWIND AND GOOD COUPLING FOR THE NEXT THROW FULL STRING LENGTH. BALL BEARING YO-YOS REQUIRE A TIGHT STRING LOOP AT THE BEARING AND INTERNAL BEARING LUBRICANT OF MODERATE VISCOSITY TO WEDGE THE FIRST TURNS IN A "SLEEPER" RESPONSE; ESPECIALLY IN WIDE STRING GAP YO-YOS.

SLACK REWIND, RESULTING IN LOSS OF STRING COUPLING IN THE NEXT THROW, CAUSES SIGNIFICANT LOSS OF INITIAL "SLEEPER" ANGULAR VELOCITY AND SPIN DURATION.

A ROUGH CHECK ON STRING COUPLING CAN BE MADE WITH ANY YO-YO: 1. HOLD THE YO-YO LOOSELY IN ONE HAND (STRING FULLY WOUND) AND ALLOW IT TO TURN AS YOU PULL THE STRING UNDER SOME TENSION TO UNWIND IT. 2. WHEN ABOUT $\frac{2}{3}$ OF THE STRING IS UNWOUND, RELEASE THE STRING AND REGRASP IT AT THE YO-YO RIM; THEN CONTINUE TO PULL IT SLOWLY WITH SOME TENSION. 3. WHERE THE STRING BEGINS TO SLIP FREE, PULL IT ALL THE WAY OUT AND ESTIMATE THE UNCOUPLED LENGTH.

MONOGRAPH IV, PAGE 20, GRAPHS STRING LENGTH AGAINST RPM_{Eof} IN A GRAVITY ONLY FALL; THE SLOPE OF THE LINE IS NEAR 6% RPM LOSS FOR EACH 10% OF ($S = 1.0m \approx 40in$) UNCOUPLED STRING. IN AN ENERGETIC "SLEEPER" THROW WHERE RPM_s ABOUT 5000 IS ESTIMATED (BY GRAPH OR BY CALCULATION), UNCOUPLING AT 10% SUGGESTS A LOSS OF AS MUCH AS 300 RPM.

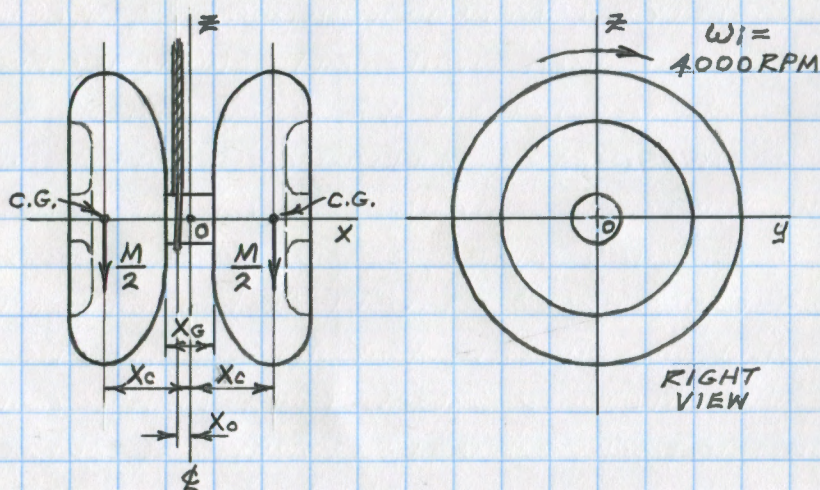
TURNING PRECESSION

IN THE OLD FIXED AXLE WOOD YO-YO DAYS, A 10+ SECOND "SLEEPER" WAS AN ACCOMPLISHMENT TO MOST PLAYERS. SKILL WAS NEEDED FOR AN ACCURATE THROW (GIVEN THE NARROW $1/16"$ STRING GAPS OF THE TIMES) TO KEEP SPINNING SIDES FREE OF THE STRING; YO-YO PRECESSION (TILTING OR TURNING) WERE SELDOM NOTICED IN THE VERY BRIEF SPIN DURATIONS. TIMES HAVE CHANGED.

BALL-BEARING AXLE YO-YOS, POPULAR SINCE THE EARLY 1990'S, ALLOW PLAYERS MUCH LONGER SPINS; WITH NOVICES ACHIEVING 60 SECONDS, AND EXPERTS MANY MINUTES. STRING GAPS MOVED UP TO ABOUT $3/32"$. PLAYERS SOON OBSERVED SLOW TILTING OF THE YO-YO IN A LONG SPIN, BRINGING A SPINNING RIM TOWARD CONTACT WITH THE STRING. SOME PLAYERS LEARNED THAT TWISTING THE STRING WITH FINGERS AN INCH OR TWO ABOVE THE YO-YO COULD STOP THE TILTING AND REVERSE ITS DIRECTION. SEE MONOGRAPH IV, PAGE 30, FOR FURTHER DISCUSSION.

TIME MARCHES ON. IN JUST THE PAST FEW YEARS, STRING GAPS AT $3/16"$ (AND WIDER!) HAVE APPEARED IN POPULAR YO-YOS NOW APPROACHING WEIGHTS OF 70 TO 90 GRAMS OR MORE. THIS COMBINATION INTRODUCES SIGNIFICANT (EASILY NOTICED) TURNING OF THE SLEEPING YO-YO ABOUT THE VERTICAL STRING AXIS. THIS IS A NEW SUBJECT WORTHY OF INVESTIGATION.

ASSUME THE YO-YO SHOWN HERE TO BE SPINNING AT AN INITIAL "SLEEPER" ANGULAR VELOCITY $\omega_i = 4000 \text{ RPM}$ SUPPORTED WITH ITS STRING OFFSET A DISTANCE X_0 LEFT OF THE CENTER O. THE OBVIOUS WEIGHT IMBALANCE OF THE YO-YO IN ITS GYROSCOPIC SPIN CAUSES IT TO TURN ABOUT THE VERTICAL AXIS OF THE STRING. THE INITIAL RATE OF TURN CAN BE ESTIMATED AND ITS DIRECTION DETERMINED.



CENTER OF GRAVITY DISTANCE FROM THE YO-YO CENTER IS X_C FOR EACH HALF. SPIN IS CLOCKWISE ABOUT THE X-AXIS. FOR A WIDE-GAP YO-YO, LET:

$$X_G = \text{STRING GAP WIDTH, } 0.180 \text{ in}$$

$$X_C = 0.500 \text{ in, AND } X_0 = 0.050 \text{ in}$$

HERE, WITH A COMMON STRING WIDTH OF 0.050 in , STRING CLEARANCE TO THE LEFT HALF IS 0.015 in , ASSURING NO CONTACT OF A SPINNING YO-YO HALF WITH THE STATIONARY VERTICAL STRING.

THE STRING OFFSET X_0 SUBJECTS THE YO-YO TO A NET CLOCKWISE TORQUE T_N . COUNTER-CLOCKWISE AND CLOCKWISE TORQUES T_{cc} AND T_c ARE, FROM THE YO-YO SKETCH:

$$T_{cc} = \frac{M}{2}(X_c - X_0); \quad T_c = \frac{M}{2}(X_c + X_0)$$

$$T_N = T_c - T_{cc} = \frac{M}{2}[(X_c + X_0) - (X_c - X_0)]$$

$$T_N = MX_0$$

TURN RATE. USING $M = 45 \times 10^{-3} \text{ kg}$ GIVEN FOR THE BÜRGER (PLAYABLE) YO-YO, THE CLOCKWISE TORQUE T_N AND ITS TORQUE VECTOR \vec{T}_N CAN EACH BE DETERMINED AS:

$$T_N = MX_0 = 45 \times 10^{-3} \text{ kg} \cdot 0.050 \text{ m} \times 25.4 \times 10^{-3} \frac{\text{m}}{\text{m}}$$

$$T_N = 57.2 \times 10^{-6} \text{ kg-m}$$

T_N AS A VECTOR (IN Newton-meters) IS:

$$\vec{T}_N = 57.2 \times 10^{-6} \text{ kg-m} \cdot \frac{1 \text{ N}}{0.102 \text{ kg}} \approx 561 \times 10^{-6} \text{ N-m}$$

$$\vec{T}_N = 561 \times 10^{-6} \frac{\text{kg-m}^2}{\text{sec}^2}$$

THE BÜRGER (PLAYABLE) YO-YO WAS ASSIGNED $I = 28125 \times 10^{-9} \text{ kg-m}^2$ AND WE HAVE ASSUMED IT TO BEGIN ITS "SLEEPER" AT 4000 RPM; CONVERTED TO radians/sec, ω BECOMES:

$$\omega_i = 4000 \frac{\text{rev}}{\text{min}} \cdot 2\pi \frac{\text{rad}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 419 \frac{\text{rad}}{\text{sec}}$$

THE SPINNING YO-YO DEVELOPS AN INITIAL ANGULAR MOMENTUM VECTOR M_i :

$$M_i = I\omega_i = 28125 \times 10^{-9} \text{ kg-m}^2 \cdot 419 \frac{\text{rad}}{\text{sec}}$$

$$M_i = 11.8 \times 10^{-3} \frac{\text{kg-m}^2}{\text{sec}}$$

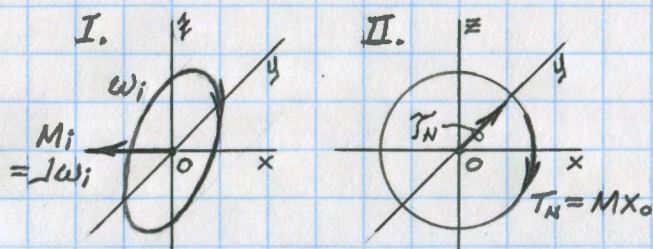
THE INITIAL GYROSCOPIC TURN RATE Ω_i IS:

$$\Omega_i = \frac{\gamma_N}{M_i} = \frac{561 \times 10^{-6} \frac{\text{kg-m}^2}{\text{sec}^2}}{11.8 \times 10^{-3} \frac{\text{kg-m}^2}{\text{sec}}} \approx 4.8 \times 10^{-3} \frac{\text{rad}}{\text{sec}} \times \frac{360 \text{ deg}}{2\pi \text{ rad}}$$

$$\Omega_i = 2.8 \frac{\text{deg}}{\text{sec}}$$

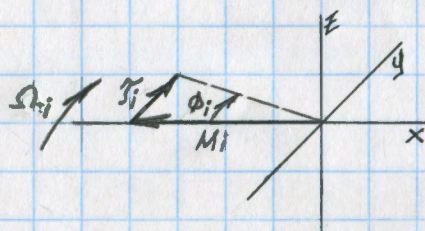
IN MONOGRAPH IV, PAGE 36, THE TILTING RATE OF THIS YO-YO IN A 4000 RPM "SLEEPER", CAUSED BY STRING TWIST TORQUE, WAS CALCULATED TO BE 0.9 deg/sec. WE NOW SEE THAT TURN RATES CAN BE SIGNIFICANTLY GREATER IN THE SAME YO-YO. PRECESSION TILTING ROTATES THE VERTICAL AXIS; OF THE YO-YO; PRECESSION TURNING ROTATES THE YO-YO ABOUT THE VERTICAL AXIS. THE DIRECTION OF TURN CAN BE PREDICTED.

TURN DIRECTION. MONOGRAPH IV, PAGE 37, GAVE SKETCHES OF "VECTOR CARDINAL DIRECTIONS" FOR A BODY SUBJECTED TO TORQUE T OR SPINNING AT ANGULAR VELOCITY ω . SKETCHES I AND II ARE ADAPTED FOR USE HERE.



THE Y_0 - Y_0 SPINS IN THE y - z PLANE ABOUT THE x -AXIS. THE NET CLOCKWISE TORQUE T_N IS EXERTED IN THE x - z PLANE ABOUT THE y -AXIS.

USING THE "RIGHT-HAND RULE" (FINGERS CURLED AROUND THE AXIS OF ROTATION), THE THUMB EXTENDS IN THE DIRECTION OF AN ANGULAR MOMENTUM VECTOR FOR $M_i = J\omega_i$. SIMILARLY, FOR TORQUE USING THE "RIGHT HAND RULE", THE THUMB EXTENDS IN THE DIRECTION OF A TORQUE VECTOR FOR T_N . VECTORS M_i AND T_N , BOTH IN THE x - y PLANE, JOIN TO DEFINE THE DIRECTION OF Y_0 - Y_0 TURNING PRECESSION.



IN A BRIEF INTERVAL AT THE START OF A "SLEEPER" SPIN, M_i AND T_N INITIATE THE Y_0 - Y_0 TURN THROUGH A SMALL INITIAL ANGLE ϕ_i . IN THAT BRIEF INSTANT, THE VECTOR-INDUCED TURN RATE IS Ω_i deg/sec; IN THIS CASE, IN THE CLOCKWISE DIRECTION ABOUT THE z -AXIS AS SHOWN.

AS THE "SLEEPER" PROGRESSES, T_N AND ITS COMPANION VECTOR J_N ARE CONSTANT, BUT ω_i AND ITS COMPANION VECTOR M_i STEADILY DECREASE. GIVEN

THE EQUATION $\Omega_i = I_A/M_i$; deg/sec, THE CALCULATED $\Omega_i = 2.8$ deg/sec FOR THIS Y_0-Y_0 AT $\omega_i = 4000$ RPM IS VALID ONLY AT THE START OF THE "SLEEPER". AS THE Y_0-Y_0 LOSES SPINNING ANGULAR VELOCITY AND MOMENTUM $I\omega_i$, THE TURN RATE INCREASES.

TURN RATE CONTROL. STRING OFFSET X_0 AND RADIUS OF GYRATION k_0 . EACH HAVE VERY SIGNIFICANT EFFECT ON Y_0-Y_0 PRECESSION TURN RATE. IN THE FOLLOWING ANALYSIS, THE SYMBOL " \propto " IS TAKEN TO MEAN "VARIES AS", A USEFUL SUBSTITUTE FOR THE "=" SYMBOL - AVOIDING ARITHMETIC. WE CAN WRITE:

$$\Omega_i \propto \frac{I_A}{M_i} \propto \frac{M X_0}{I \omega_i}; I = M k_0^2$$

EARLIER, Y_0-Y_0 ANGULAR VELOCITY ω_i rad/sec. AND RPM_{EOP} WERE FOUND TO VARY INVERSELY AS k_0 ; IN A "SLEEPER", $\omega_i \propto 1/k_0$. REPLACING I AND ω_i :

$$\Omega_i \propto \frac{M X_0}{M k_0^2 \frac{1}{k_0}} \propto \frac{X_0}{k_0}$$

$\Omega_i \propto X_0/k_0$ DEFINES THAT INITIAL "SLEEPER" TURN RATE VARIES DIRECTLY WITH STRING OFFSET AND INVERSELY WITH RADIUS OF GYRATION.

IT HAS BEEN CLEAR THAT STRING OFFSET CAN CAUSE HIGH TURN RATE. IT MAY NOT HAVE BEEN EQUALLY CLEAR THAT INCREASING Y_0-Y_0 RADIUS OF GYRATION CAN REDUCE TURN RATE; CENTERING THE STRING IN THE STRING GAP WORKS WELL ALSO.

SUMMARY

FRICTION GENERATED ENERGY LOSSES IN A FREE FALL OR THROWN "SLEEPER" ARE IGNORED; NO CONTACT BETWEEN UNWOUND STRING AND SPINNING RIM IS ASSUMED AND ENERGY LOSS TO RESILIENT STRING STRETCHING IS ASSUMED MINOR. HOWEVER, FRICTIONAL COUPLING OF WOUND STRING TO SPINNING SIDES IS ASSUMED FULLY ACTIVE FOR THE EFFECTIVE STRING LENGTH ($S=1.0\text{m}$). ACTUAL RPM VALUES WILL THEREFORE ALWAYS BE LESS THAN THOSE CALCULATED OR GRAPHED; SEE ESPECIALLY PAGE 34.

SALIENT PRINCIPLES FROM THE MONOGRAPH I-IV STUDIES INCLUDE:

- INCREASING RADIUS OF GYRATION k_0 BY "RIM-WEIGHTING" A YO-YO PROFILE INCREASES "SLEEPER" MOMENTUM, GYROSCOPIC STABILITY, AND SPIN DURATION, BUT REDUCES INITIAL "SLEEPER" RPM.
- WEIGHT RINGS WITH RADIUS OF GYRATION LESS THAN THAT OF A HOST YO-YO RESULT IN A COMPOSITE k_0 LESS THAN THAT OF THE HOST; RING RADIUS OF GYRATION GREATER THAN THAT OF THE HOST RESULTS IN A COMPOSITE k_0 GREATER THAN THAT OF THE HOST.
- INITIAL "SLEEPER" ENERGY OF ROTATION CANNOT EXCEED THE ENERGY SUM FROM VERTICAL VELOCITY OF TRANSLATION AS THE YO-YO LEAVES THE HAND AND GRAVITY ACCELERATION FOR STRING LENGTH ($S=1.0\text{m}$).
- RADIUS OF GYRATION k_0 IS THE MOST IMPORTANT SINGLE PHYSICAL VARIABLE FOR THE YO-YO.
- INVESTIGATING YO-YO PHYSICS IS FUN.

THIS MONOGRAPH SERIES WAS CONCEIVED MORE THAN A DECADE AGO WHEN I FIRST LEARNED OF WOLFGANG BÜRGER'S "THE YO-YO: A TOY FLYWHEEL" (SEE REFERENCES). MANY LIBRARY AND INTERNET SEARCHES YIELDED LITTLE IN-DEPTH STUDY OF YO-YO PHYSICS TO COMPARE WITH THAT ARTICLE. I'M INDEBTED TO PROFESSOR BÜRGER FOR HIS INSPIRATION; USING HIS ARTICLE AS A BASE, THE SERIES HAS TAKEN FOUR YEARS TO COMPLETE WITH ONLY SIMPLE TOOLS.

THE YOMEGA "RPM" YO-YO ($k_0 \approx 0.75$ in) DISPLAYS "SLEEPER" INITIAL (MAXIMUM) RPM TO FOUR DIGITS. MY BEST THROWS REGISTER ABOUT 7000 RPM GIVING ME A "SLEEPER" THROW VERTICAL VELOCITY OF ABOUT 30 mi/hr (SEE PAGES 27-29). NO RESPECTABLE PHYSICS CLASSROOM OR LABORATORY SHOULD BE WITHOUT THIS GREAT TOOL.

A "SLEEPER"-THROWING MECHANICAL ROBOT LIKE THE "IRON BYRON" FOR GOLF STICK AND BALL TESTING MIGHT BE USEFUL. HIGH SPEED STROBE PHOTOGRAPHY CAN ACCURATELY ANALYZE "SLEEPER" THROW VELOCITY, INITIAL "SLEEPER" RPM, AND LOSS OF STRING COUPLING. ALL WAY BEYOND MY RESEARCH AND ENGINEERING BUDGET.

MANY THANKS TO MANY YO-YO DESIGNERS, MANUFACTURERS, MARKETERS, SKILLED PLAYERS, MODIFIERS, AND ALL THOSE OTHER KIDS FOR ALL THE FUN IN BOTH YO-YO PLAY AND STUDY IN THE '30s, EARLY '40s, AND PAST 16 YEARS.

HAPPY YO-YO DAYS TO ALL-

(NW) 9/26/03

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- MONOGRAPH II. THE ACADEMIC YO-YO

- MONOGRAPH III. THE SLEEPING YO-YO

- MONOGRAPH IV. MECHANICS AND GYROSCOPICS



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